

Lec 2A:

04/05/2017

## Thin-Disk Spectrum:

In order to calculate the disk spectrum, we first need to know whether it is optically thin or thick. The optical depth in the perpendicular direction is:

$$\tau_{\perp} = \int_{-\infty}^{+\infty} n_e(z) \sigma_T dz = \frac{\sigma_T}{\mu m_H} \int_{-\infty}^{+\infty} \rho(z) dz = \frac{\sigma_T}{\mu m_H} \Sigma$$

Using the values of  $\sigma_T$  and  $m_H$  (proton mass), and the expression for  $\Sigma$  in terms of the disk thickness (which we found in the previous lecture), we find that  $\tau_{\perp} \sim 10^3$ . As a result, the geometrically thin disk is optically thick, which implies that the emission must be blackbody. Therefore, the

dissipation rate from the disk follows:

$$D_{(CR)} = \sigma T_{(CR)}^4 \quad (\sigma: \text{Stefan-Boltzmann Constant})$$

After using the expression that we derived for  $D_{(CR)}$  last time,

we find:

$$T(R) = \left[ \frac{3GM\dot{M}}{8\pi R^3 a} \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right] \right]^{1/4}$$

Over most of the disk  $R \gg R_*$ , and hence  $T(R) \propto R^{-3/4}$ . The intensity of radiation at radius  $R$  is:

$$I_\nu(R) = \frac{2 \frac{h\nu^3}{c^2}}{\exp\left[\frac{h\nu}{kT(R)}\right] - 1}$$

An observer at a distance  $D$  away, with a line of sight making an angle  $i$  relative to the symmetry axis of the disk, will measure:

$$F_\nu = \int_{R_*}^{R_{out}} I_\nu d\Omega(R)$$

Where:

$$d\Omega(R) = \frac{2\pi R dR \cos i}{D^2}$$

Thus:

$$F_\nu = \frac{4\pi h\nu^3 \cos i}{c^2 D^2} \int_{R_*}^{R_{out}} \frac{R dR}{\exp\left[\frac{h\nu}{kT(R)}\right] - 1}$$

As usual, it is useful to find the asymptotic behavior of

$F_\nu$ . In the Rayleigh-Jeans limit,  $h\nu \ll kT_{(R_{out})}$ , which implies that:

$$F_\nu^{RJ} \propto \nu^2$$

In the Wien limit, defined as  $h\nu \gg kT_{(R_*)}$ , we have:

$$F_\nu^W \propto \nu^3 \exp\left[-\frac{h\nu}{kT_{(R_*)}}\right]$$

In this limit, the flux is dominated by the hottest portion of the disk near  $R_*$ . In between the two limits, we have:

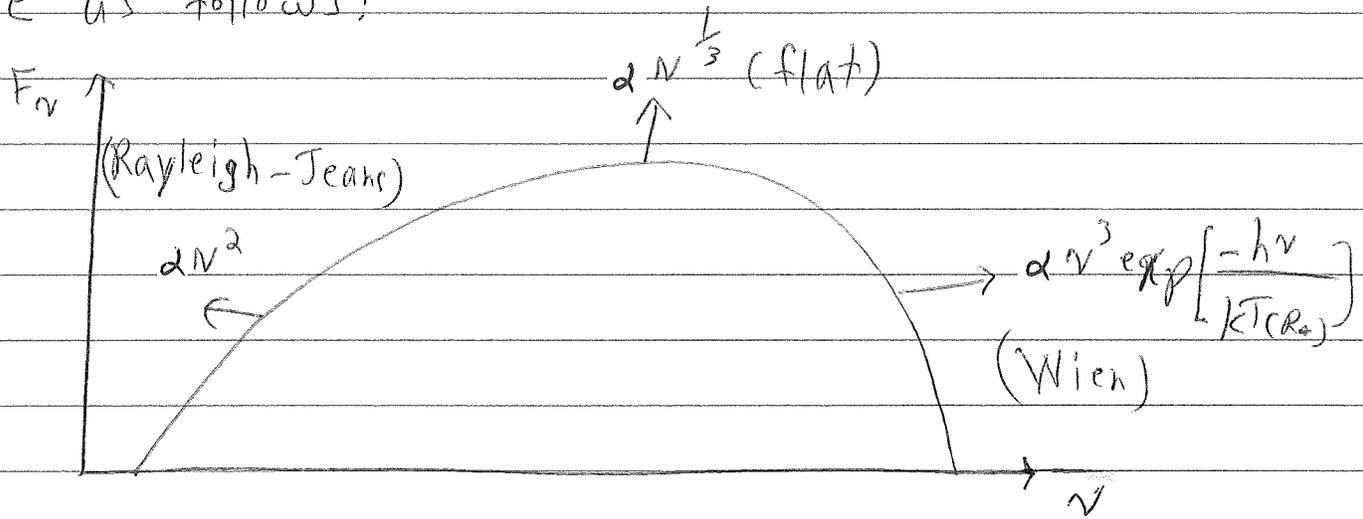
$$F_\nu \propto \nu^{\frac{1}{3}} \int_0^\infty \frac{\eta^{\frac{5}{3}}}{e^\eta - 1} d\eta \propto \nu^{\frac{1}{3}} \quad \left(\eta \equiv \frac{h\nu}{kT_{(R)}} = \frac{h\nu}{kT_{(R_*)}} \left(\frac{R}{R_*}\right)^{\frac{3}{4}}\right)$$

In the Rayleigh-Jeans and Wien limits, the spectrum is that of a single black body, although at different temperatures,  $T_{(R_{out})}$  and  $T_{(R_*)}$  respectively. Between the two limits,

the spectrum is the sum of black bodies, which results in a

(4)

segment  $\propto \nu^{1/3}$  (so-called "flat" segment). The spectrum looks like as follows:



It can be distinguished from those of other thermal sources by its stretched out appearance. It is usually not difficult to identify a spectrum belonging to a thin disk once it has been assembled from multi-frequency observations. We note that the Wien region gives information about  $T(R_*)$ , while the Rayleigh-Jeans region contains information about  $T(R_{out})$  hence  $R_{out}$ , in the transition to the middle flat region,

### Boundary Layers,

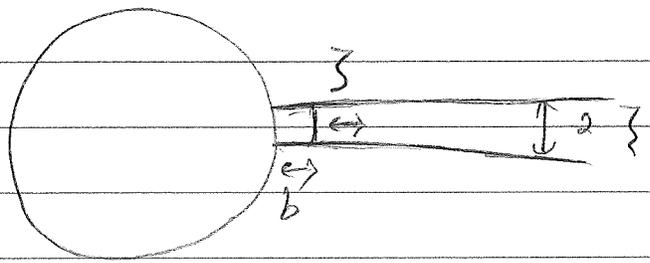
Let us now study the role played in high-energy astrophysics by the transition region between the inner edge of the disk and the surface of the compact object in more detail.

Since  $v_R \ll v_\phi$ , the angular velocity  $\Omega(R)$  in the disk remains very close to its Keplerian value until the matter reaches just outside the surface of the star at  $R=R_*$ .

Within a boundary layer of radial extent  $b$ ,  $\Omega$  must decrease from  $\Omega_K(R_*)$  to  $\Omega(R_*) < \Omega_K(R_*)$ . For a very slow rotation (as compared with the Keplerian velocity) at  $R_*$ , the boundary layer must be in hydrostatic equilibrium in the radial direction:

$$\frac{1}{\rho} \frac{\partial \rho}{\partial R} \approx -\frac{GM}{R_*^2}$$

$\rho \sim c_s^2 \rho$



Thus:

$$\frac{c_s^2}{b} \sim \frac{GM}{R_*^2}$$

We also have hydrostatic equilibrium in the vertical direction <sup>on</sup> (which was discussed last time):

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{GMz}{R_*^3} \Rightarrow \frac{P}{\rho z} \sim \frac{GMz}{R_*^3}$$

We therefore find:

$$z \sim c_s R_* \left( \frac{R_*}{GM} \right)^{1/2} \Rightarrow b \sim \frac{z^2}{R_*} = \left( \frac{z}{R_*} \right)^2 R_* \ll R_*$$

The total luminosity produced by the disk is:

$$L_d = 2 \int_{R_*}^{R_{out}} P(R) 2\pi R dR = \frac{3GM\dot{M}}{2} \int_{R_*}^{R_{out}} \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right] \frac{dR}{R^2}$$

$$\Rightarrow L_d \approx \frac{GM\dot{M}}{2R_*}$$

On the other hand, the total gravitational energy released in the accretion is:

$$L_{acc} \approx \frac{GM\dot{M}}{R_*}$$

This implies that the power emitted from transition at the

boundary layer is actually half of the total available power:

$$L_b = L_{acc} - L_d \approx \frac{GM\dot{M}}{2R_*}$$

The radiation emitted by the boundary layer is usually in the form of X-ray. It emerges through a region of radial extent  $\sim \zeta$  on the two disk faces. This results in a black body radiation with temperature  $T_b$ , where:

$$4\pi R_* \zeta \sigma T_b^4 \approx \frac{GM\dot{M}}{2R_*}$$

This results in:

$$T_b \approx \left( \frac{GM\dot{M}}{8\pi R_*^2 \zeta \sigma} \right)^{\frac{1}{4}} \approx \left( \frac{R_*}{\zeta} \right)^{\frac{1}{4}} T_{(R_*)}$$

After using the relation for  $\zeta$ , which we obtained in the previous lecture, we find:

$$T_b \approx \left[ \frac{T_0}{T_{(R_*)}} \right]^{\frac{1}{8}} T_{(R_*)} \quad \left( T_0 \equiv \frac{GM \omega_{KH}}{k R_*} \right)$$

In a neutron star:

$$M \approx M_{\odot}, R_{*} \approx 10 \text{ km} \Rightarrow T_{\circ} \approx 3 \times 10^{11} \text{ K}$$

Hence:

$$T_b \approx 3.6 \times T(R_{*}) \approx 3 \times 10^7 \text{ K}$$

At this temperature, the boundary layer can produce a significant blackbody component above the regular disk spectrum. The combination of a stretched out thin-disk spectrum plus a single (and hotter) blackbody component is indeed seen often in low-mass X-ray binaries.